

Manipulatives progression and calculation policy

Calculation Policy

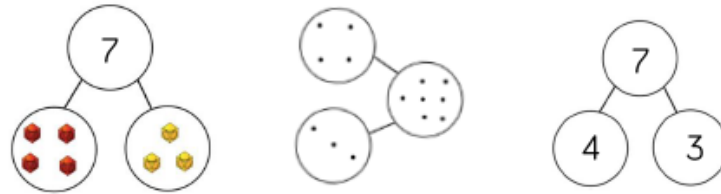
Priory's calculation policy has been structured using the key written methods and manipulatives featured within the White Rose Scheme of Learning. This is to ensure that staff and pupils are secure in the small steps which build up to the formal and efficient written methods of calculation across all four operations. It is also to ensure consistency and continuity as pupils move through school.

The abstract representations appropriate to the stage of learning the child is working at should be modelled alongside the practical representation being modelled in order to create links and develop schema within the long term memory.

The use of manipulatives is promoted throughout school with the aim of promoting independent learners. Children work through the CPA section of the lesson with guidance from teachers and teaching assistants, moving away from resources when they are confident. They may return to the manipulatives or pictorial representations as a checking strategy at any point in the lesson.



Part-Whole Model

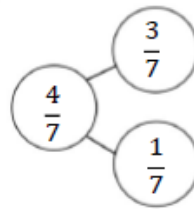
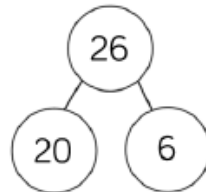
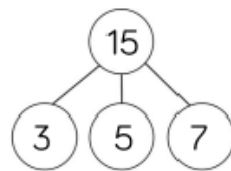


$$7 = 4 + 3$$

$$7 = 3 + 4$$

$$7 - 3 = 4$$

$$7 - 4 = 3$$



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

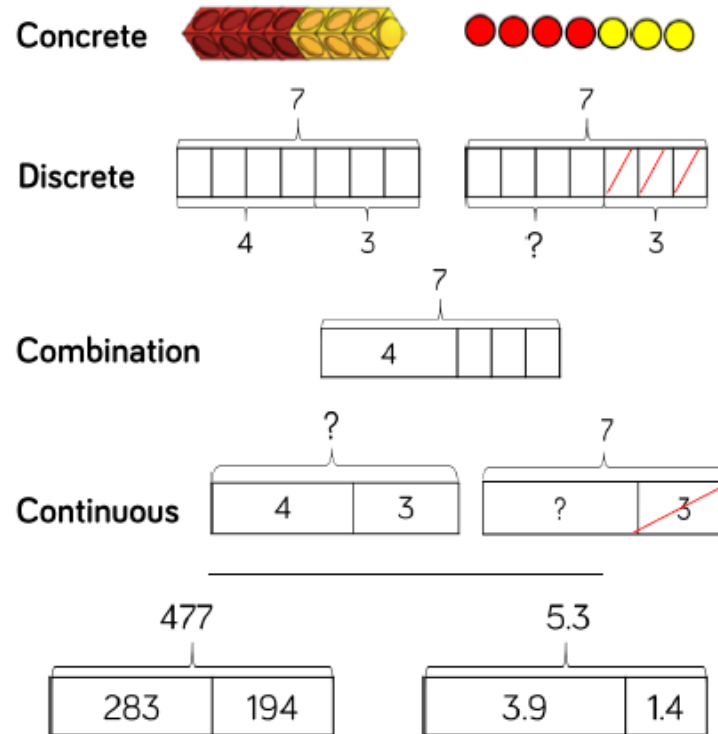
When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Bar Model (single)



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

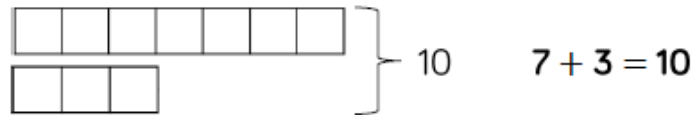
The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

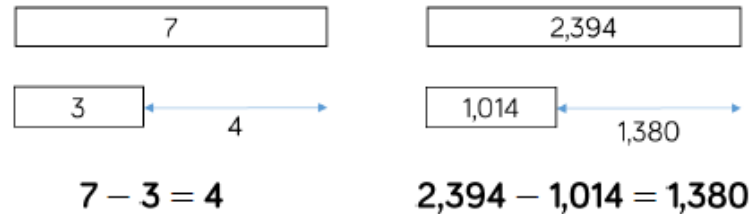
In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Bar Model (multiple)

Discrete



Continuous



Benefits

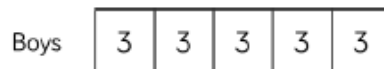
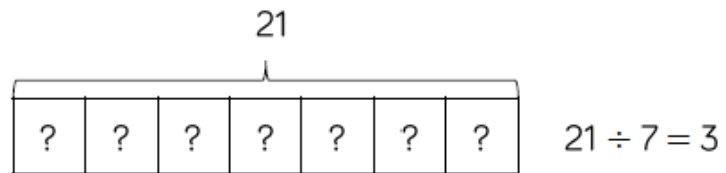
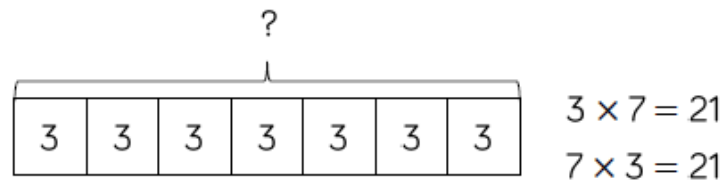
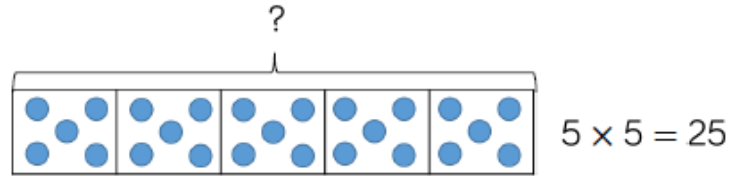
The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Bar Model



Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?

The multiple bar model provides an opportunity to compare the groups.

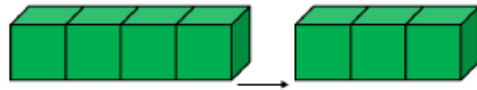
Cubes



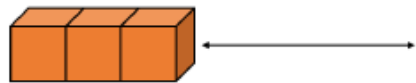
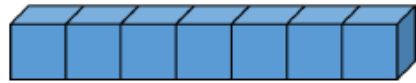
$$7 = 4 + 3$$



$$7 = 3 + 4$$



$$7 - 3 = 4$$



$$7 - 3 = 4$$

Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

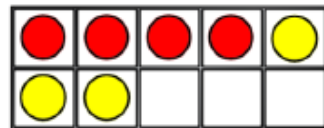
When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

Ten Frames (within 10)



$$4 + 3 = 7$$

4 is a part.

$$3 + 4 = 7$$

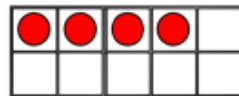
3 is a part.

$$7 - 3 = 4$$

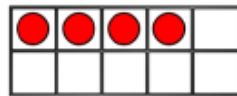
7 is the whole.

$$7 - 4 = 3$$

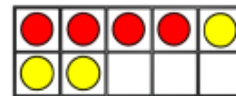
First



Then

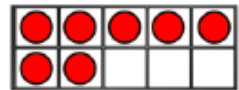


Now

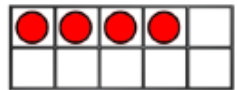


$$4 + 3 = 7$$

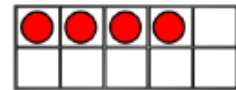
First



Then



Now



$$7 - 3 = 4$$

Benefits

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

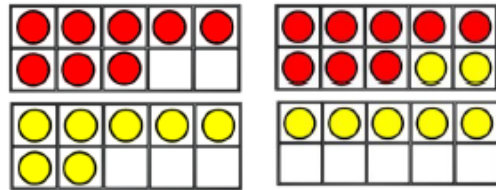
Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.

Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts.

Using these structures, the ten frame can enable children to find all the number bonds for a number.

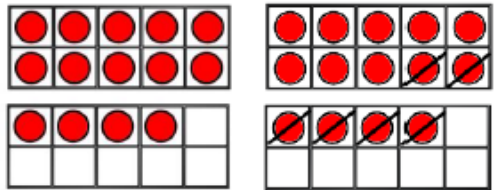
Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

Ten Frames (within 20)



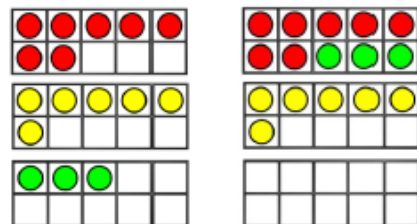
$$8 + 7 = 15$$

2 5



$$14 - 6 = 8$$

4 2



$$7 + 6 + 3 = 16$$

10

Benefits

When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

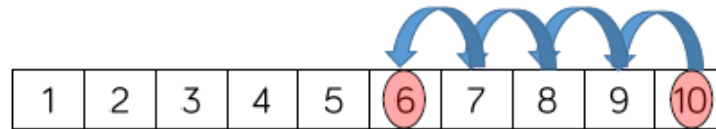
When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

Number Tracks

$$5 + 3 = 8$$



$$10 - 4 = 6$$



$$8 + 7 = 15$$



Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

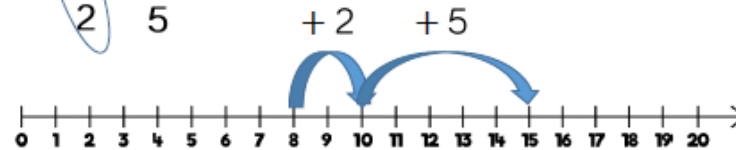
Number Lines (labelled)

$$5 + 3 = 8$$



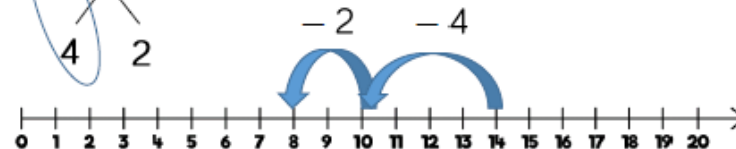
$$8 + 7 = 15$$

$$\begin{array}{r} 8 \\ + 7 \\ \hline 2 \quad 5 \end{array}$$



$$14 - 6 = 8$$

$$\begin{array}{r} 14 \\ - 6 \\ \hline 4 \quad 2 \end{array}$$



Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

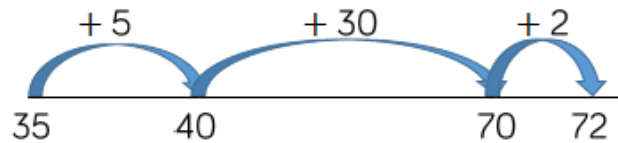
Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

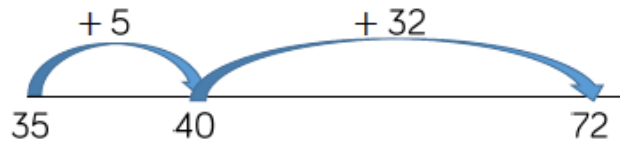
Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

Number Lines (blank)

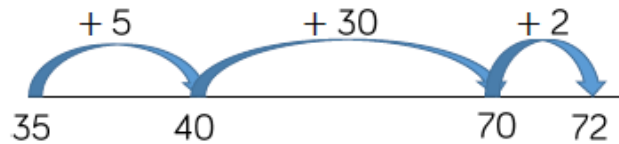
$$35 + 37 = 72$$



$$35 + 37 = 72$$



$$72 - 35 = 37$$



Benefits

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Number Tracks



$$6 \times 3 = 18$$

$$3 \times 6 = 18$$



$$18 \div 3 = 6$$

Benefits

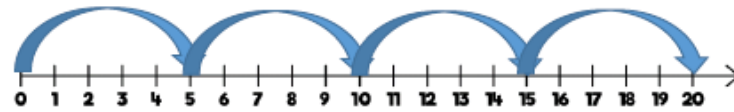
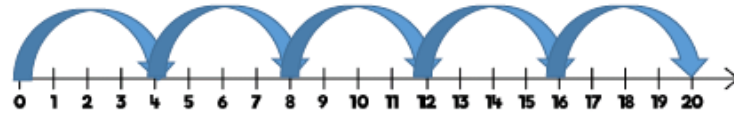
Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.

When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

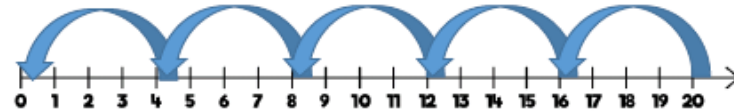
Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

Number Lines (labelled)



$$4 \times 5 = 20$$

$$5 \times 4 = 20$$



$$20 \div 4 = 5$$

Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

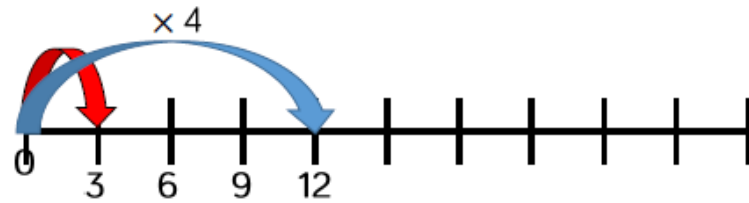
When multiplying, children start at 0 and then count on to find the product of the numbers.

When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.

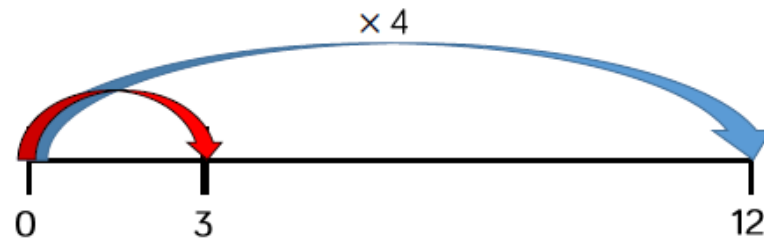
Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

Number Lines (blank)



A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

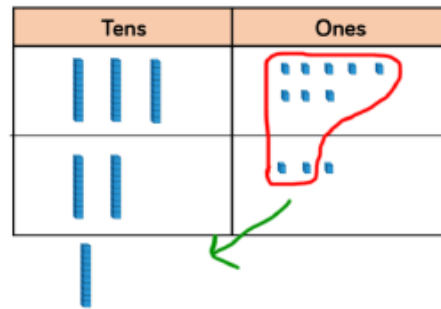
Benefits

Children can use blank number lines to represent scaling as multiplication or division.

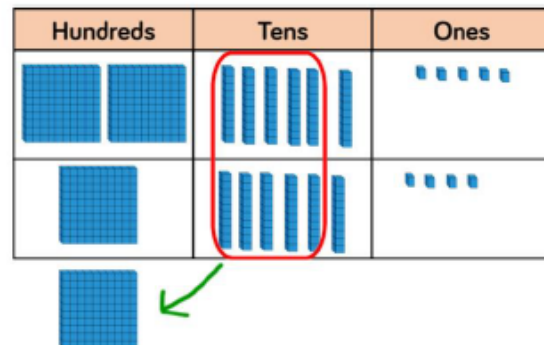
Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.

Base 10/Dienes (addition)



$$\begin{array}{r} 38 \\ + 23 \\ \hline 61 \\ \hline 1 \end{array}$$



$$\begin{array}{r} 265 \\ + 164 \\ \hline 429 \\ \hline 1 \end{array}$$

Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

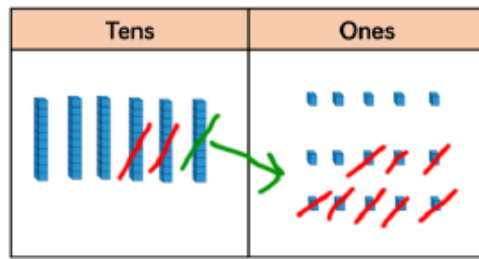
Children should first add without an exchange before moving on to addition with exchange. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children.

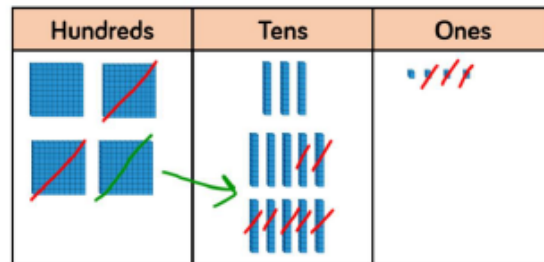
- How many ones are there altogether?
- Can we make an exchange? (Yes or No)
- How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)
- How many ones do we have left? (Write in ones column)

Repeat for each column.

Base 10/Dienes (subtraction)



$$\begin{array}{r} 5 \quad 1 \\ 65 \\ - 28 \\ \hline 37 \end{array}$$



$$\begin{array}{r} 3 \quad 1 \\ 435 \\ - 273 \\ \hline 262 \end{array}$$

Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

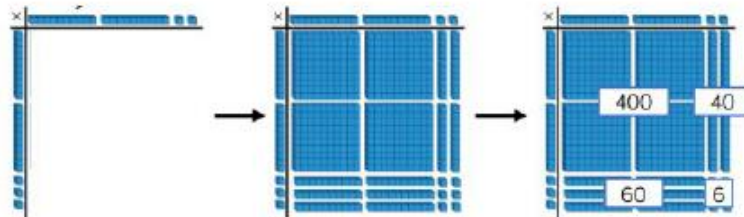
Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

Base 10/Dienes (multiplication)

Hundreds	Tens	Ones
		●●●●
		●●●●
		●●●●

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \\ \hline 1 \end{array}$$



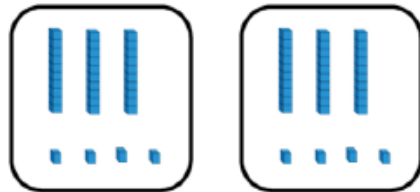
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

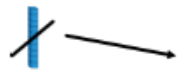
As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces. This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.

Base 10/Dienes (division)

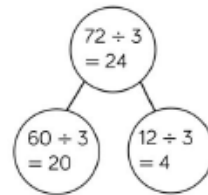


$$68 \div 2 = 34$$



Tens	Ones

$$72 \div 3 = 24$$



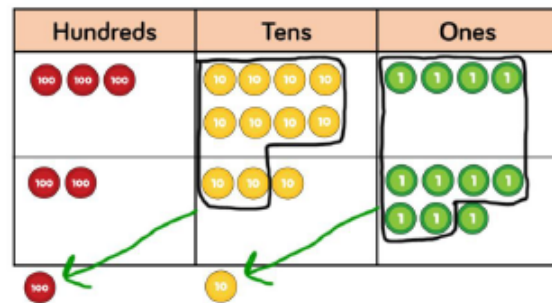
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of division.

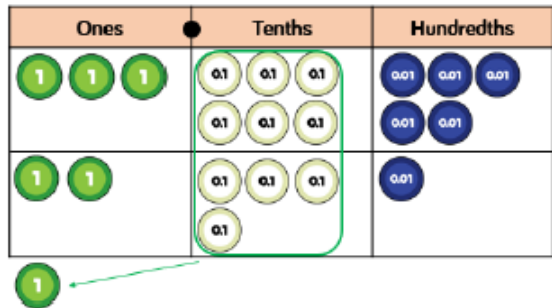
When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

Place Value Counters (addition)



$$\begin{array}{r} 384 \\ + 237 \\ \hline 621 \\ 1 \end{array}$$



$$\begin{array}{r} 3.65 \\ + 2.41 \\ \hline 6.06 \\ 1 \end{array}$$







Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.


Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Place Value Counters (Subtraction)

Hundreds	Tens	Ones
		
		

$$\begin{array}{r} \overset{4}{\cancel{6}} \overset{1}{\cancel{5}} 2 \\ - 207 \\ \hline 445 \end{array}$$

Thousands	Hundreds	Tens	Ones
			
			

$$\begin{array}{r} \overset{3}{\cancel{4}} \overset{1}{\cancel{3}} 57 \\ - 2735 \\ \hline 1622 \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

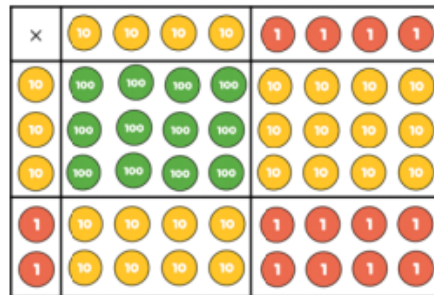
Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

Place Value Counters (multiplication)



$$\begin{array}{r} 34 \\ \times 5 \\ \hline 170 \\ \hline 12 \end{array}$$



$$\begin{array}{r} 44 \\ \times 32 \\ \hline 8 \\ 80 \\ 120 \\ + 1200 \\ \hline 1408 \\ \hline 1 \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed. The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.

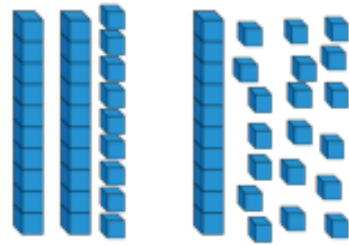


Priory
Primary School

Number and Place value

Same concept, different representations

Base 10



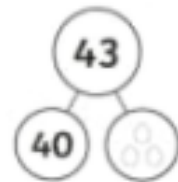
29

Place value counters



Place value grid

Tens	Ones



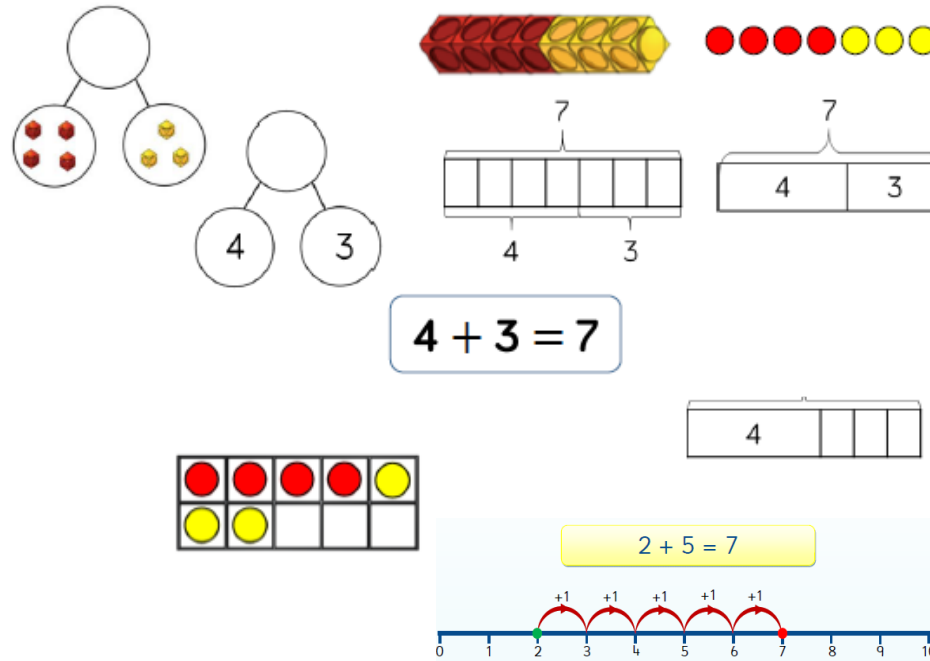


Priory
Primary School

Addition

Skill: Add 1-digit numbers within 10

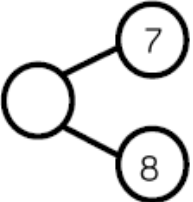
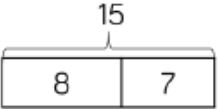
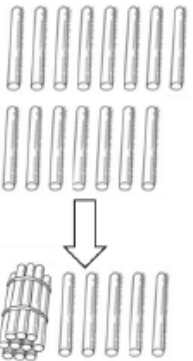
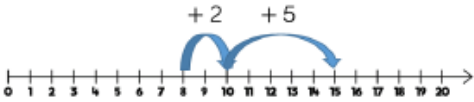
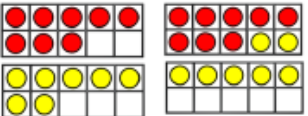
Year: 1

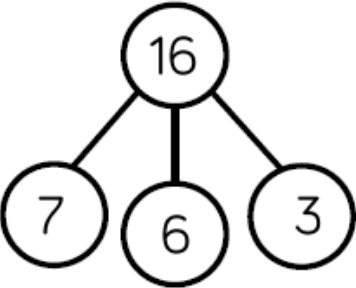
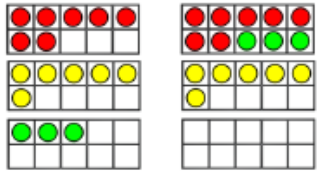
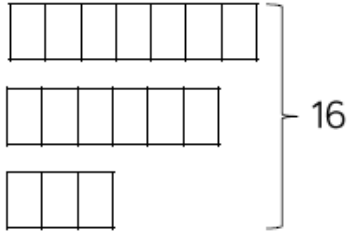


When adding numbers to 10, children can explore both aggregation and augmentation.

The part-whole model, discrete and continuous bar model, number shapes and ten frame support aggregation.

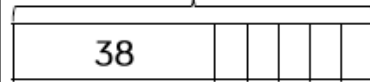
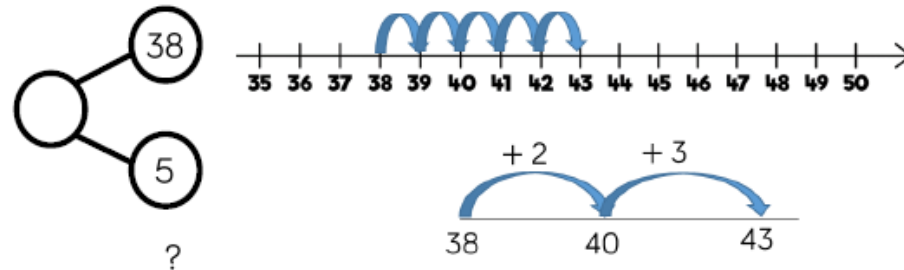
The combination bar model, ten frame, bead string and number track all support augmentation.

Skill: Add 1 and 2-digit numbers to 20	Year: 1/2
<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <div style="text-align: center; margin: 20px 0;"> $8 + 7 = 15$ </div> <div style="display: flex; justify-content: center; align-items: center;">  </div> <div style="text-align: center; margin-top: 20px;">  </div>	<p>When adding one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten.</p> <p>Different manipulatives can be used to represent this exchange. Use concrete resources alongside number lines to support children in understanding how to partition their jumps.</p>

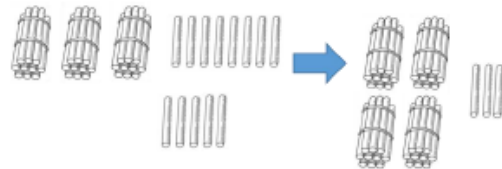
Skill: Add three 1-digit numbers	Year: 2				
<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  </div> <div style="text-align: center;"> <p>3. $6 + 1 + \square = 10$</p> <table border="1" style="margin: auto;"> <tr><td colspan="2" style="text-align: center;">10</td></tr> <tr><td style="text-align: center;">6</td><td style="text-align: center;">1</td></tr> </table> </div> </div> <div style="text-align: center; margin: 10px 0;"> <div style="border: 1px solid black; border-radius: 10px; padding: 5px; display: inline-block;">$7 + 6 + 3 = 16$</div> </div> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  </div> <div style="text-align: center;"> <p>$7 + 6 + 3 = 16$</p> <p>10</p> </div> <div style="text-align: center;">  </div> </div>	10		6	1	<p>When adding three 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently.</p> <p>This supports children in their understanding of commutativity.</p> <p>Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers.</p>
10					
6	1				

Skill: Add 1-digit and 2-digit numbers to 100

Year: 2/3



$38 + 5 = 43$



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

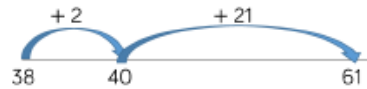
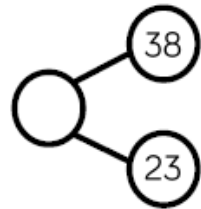
When adding single digits to a two-digit number, children should be encouraged to count on from the larger number.

They should also apply their knowledge of number bonds to add more efficiently e.g. $8 + 5 = 13$ so $38 + 5 = 43$.

Hundred squares and straws can support children to find the number bond to 10.

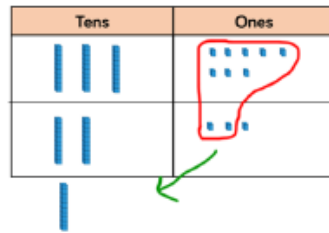
Skill: Add two 2-digit numbers to 100

Year: 2/3



?	
38	23

$$38 + 23 = 61$$



$$\begin{array}{r} 38 \\ + 23 \\ \hline 61 \\ 1 \end{array}$$

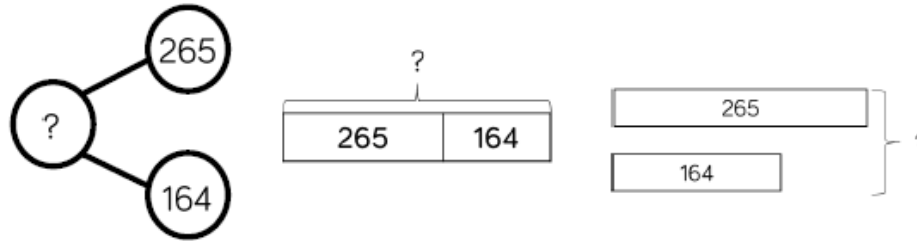


At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

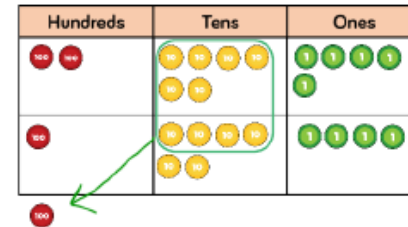
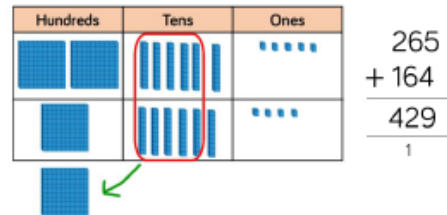
Children can also use a blank number line to count on to find the total. Encourage them to jump to multiples of 10 to become more efficient.

Skill: Add numbers with up to 3 digits

Year: 3



$$265 + 164 = 429$$



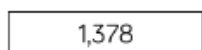
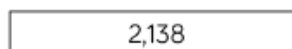
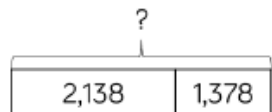
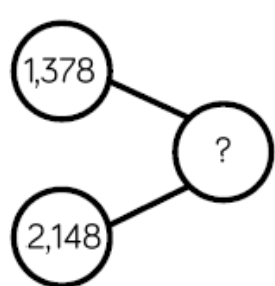
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

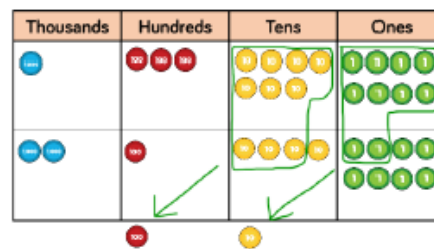
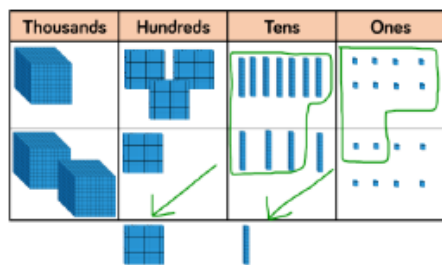
Skill: Add numbers with up to 4 digits

Year: 4



1	3	7	8				
+	2	1	4	8			
				3	5	2	6
					1	1	

$$1,378 + 2,148 = 3,526$$



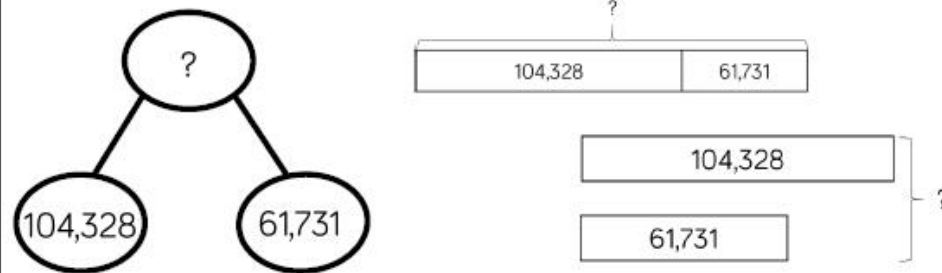
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

Skill: Add numbers with more than 4 digits

Year: 5/6



$$104,328 + 61,731 = 166,059$$

HTh	TTh	Th	H	T	O
●		●●●●	●●●●	●●	●●●●
	●●●●	●	●●●●	●●●●	●

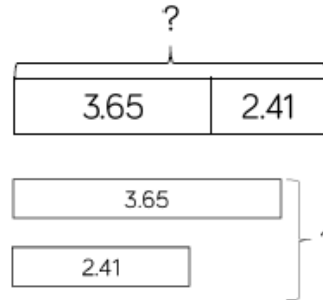
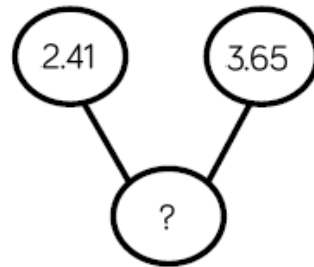
1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9
					1

Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.

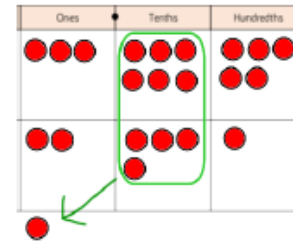
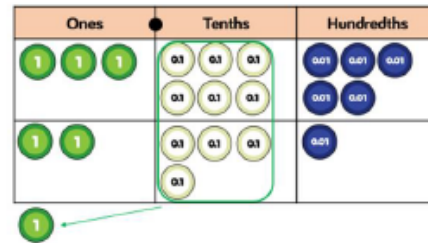
Skill: Add with up to 3 decimal places

Year: 5



$$\begin{array}{r} 3.65 \\ + 2.41 \\ \hline 6.06 \\ 1 \end{array}$$

$$3.65 + 2.41 = 6.06$$



Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.

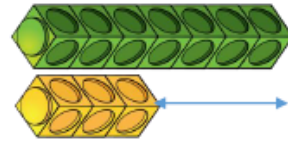
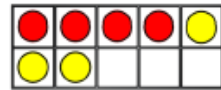
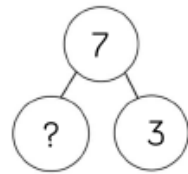


Priory
Primary School

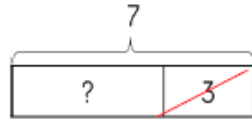
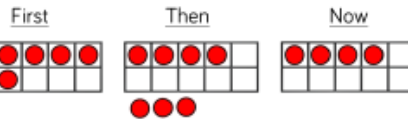
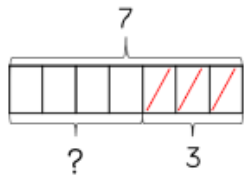
Subtraction

Skill: Subtract 1-digit numbers within 10

Year: 1



$$7 - 3 = 4$$



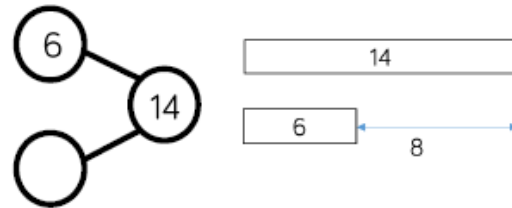
Part-whole models, bar models, ten frames and number shapes support partitioning.

Ten frames, number tracks, single bar models and bead strings support reduction.

Cubes and bar models with two bars can support finding the difference.

Skill: Subtract 1 and 2-digit numbers to 20

Year: 1/2

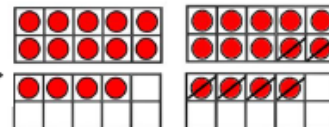
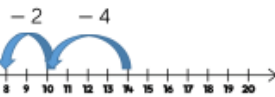


$$14 - 6 = 8$$



$$14 - 6 = 8$$

A number bond diagram with 14 in a central circle, connected to 4 and 2 in smaller circles.



$$14 - 6 = 8$$

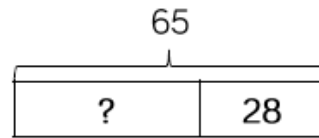
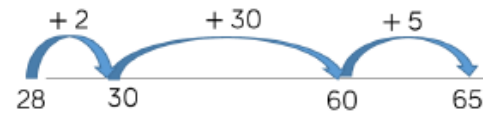
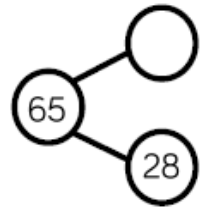
A number bond diagram with 14 in a central circle, connected to 4 and 2 in smaller circles.

When subtracting one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten.

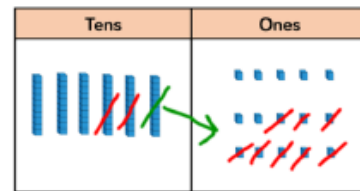
Children should be encouraged to find the number bond to 10 when partitioning the subtracted number. Ten frames, number shapes and number lines are particularly useful for this.

Skill: Subtract 1 and 2-digit numbers to 100

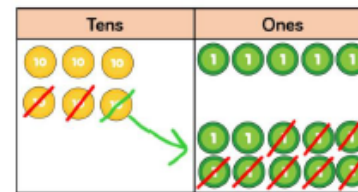
Year: 2



$$65 - 28 = 37$$



$$\begin{array}{r} 5 \ 1 \\ 65 \\ - 28 \\ \hline 37 \end{array}$$

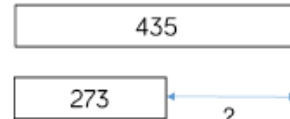
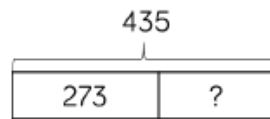
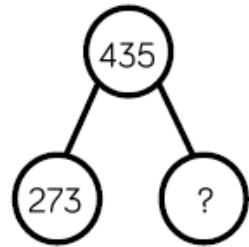


At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

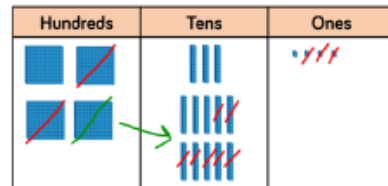
Children can also use a blank number line to count on to find the difference. Encourage them to jump to multiples of 10 to become more efficient.

Skill: Subtract numbers with up to 3 digits

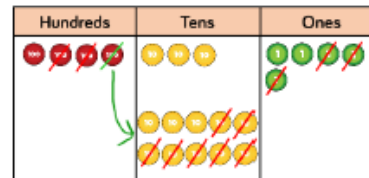
Year: 3



$$435 - 273 = 262$$



$$\begin{array}{r} 3 \quad 1 \\ 435 \\ - 273 \\ \hline 262 \end{array}$$



Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits.









Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

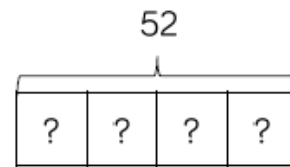
Plain counters on a place value grid can also be used to support learning.

Skill: Divide 2-digits by 1-digit (sharing with exchange)

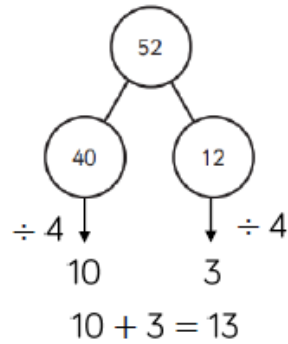
Year: 3/4









Tens	Ones
	
	
	
	



$$52 \div 4 = 13$$



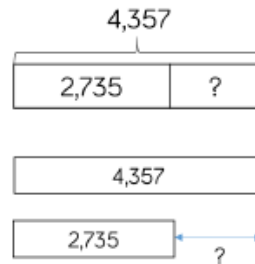
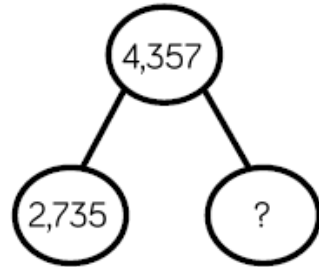
Tens	Ones
	
	
	
	

When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones. Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

Flexible partitioning in a part-whole model supports this method.

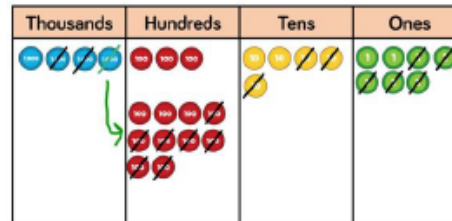
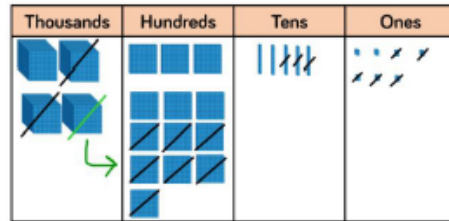
Skill: Subtract numbers with up to 4 digits

Year: 4



$$\begin{array}{r} 3 \ 1 \\ 4357 \\ - 2735 \\ \hline 1622 \end{array}$$

$$4,357 - 2,735 = 1,622$$



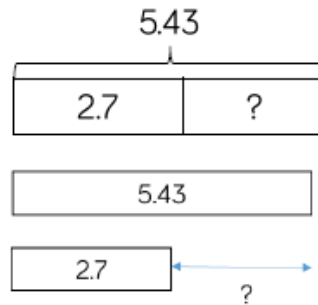
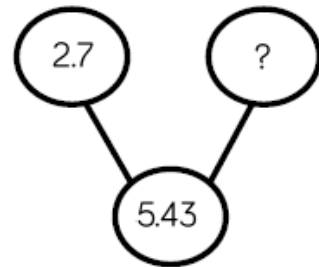
Base 10 and place value counters are the most effective manipulatives when subtracting numbers with up to 4 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

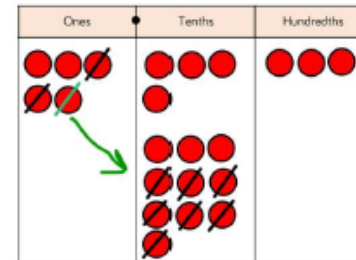
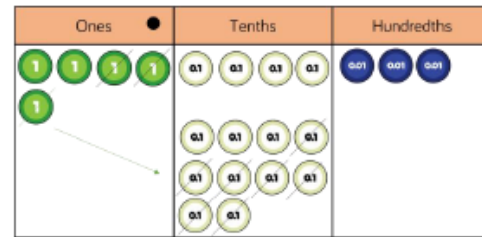
Skill: Subtract with up to 3 decimal places

Year: 5



$$\begin{array}{r} 4 \text{ } 1 \\ 5.43 \\ - 2.7 \\ \hline 2.73 \end{array}$$

$$5.43 - 2.7 = 2.73$$



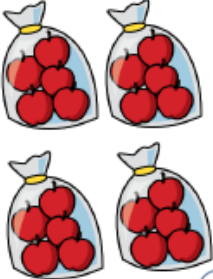
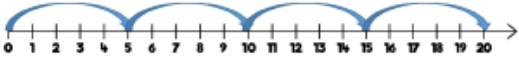
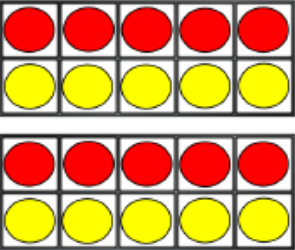
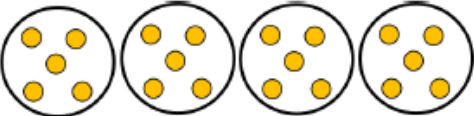

Place value counters and plain counters on a place value grid are the most effective manipulative when subtracting decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.



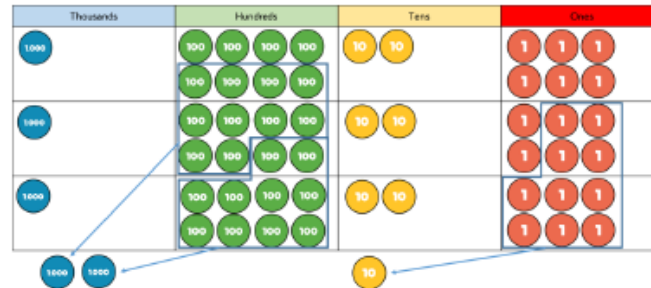
Priory
Primary School

Multiplication

Skill: Solve 1-step problems using multiplication	Year: 1/2
  <p data-bbox="958 694 1500 808">One bag holds 5 apples. How many apples do 4 bags hold?</p>    <p data-bbox="1370 972 1663 1108"> $5 + 5 + 5 + 5 = 20$ $4 \times 5 = 20$ $5 \times 4 = 20$ </p>	<p data-bbox="1696 444 1951 586">Children represent multiplication as repeated addition in many different ways.</p> <p data-bbox="1696 629 1964 879">In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally.</p> <p data-bbox="1696 922 1964 1029">In Year 2, children are introduced to the multiplication symbol.</p>

Skill: Multiply 4-digit numbers by 1-digit numbers

Year: 5



$$1,826 \times 3 = 5,478$$

	Th	H	T	O
	1	8	2	6
x				3
	5	4	7	8
	2		1	

When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method. If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method.

Skill: Multiply 2-digit numbers by 2-digit numbers

Year: 5

3

$$22 \times 31 = 682$$

	H	T	O
		2	2
x		3	1
		2	2
	6	6	0
	6	8	2

When multiplying a multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by the Base 10. The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.

Skill: Multiply 3-digit numbers by 2-digit numbers

Year: 5

$$234 \times 32 = 7,488$$

Th	H	T	O
	2	3	4
×		3	2
	4	6	8
¹ 7	¹ 0	2	0
7	4	8	8

×	200	30	4
30	6,000	900	120
2	400	60	8

Children can continue to use the area model when multiplying 3-digits by 2-digits. Place value counters become more efficient to use but Base 10 can be used to highlight the size of numbers.

Encourage children to move towards the formal written method, seeing the links with the grid method.

Skill: Multiply 4-digit numbers by 2-digit numbers

Year: 5/6

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2	1	9	1	2
₂	₅	₃	₇	
5	4	7	8	0
₁		₁		
7	6	6	9	2

1

$$2,739 \times 28 = 76,692$$

When multiplying 4-digits by 2-digits, children should be confident in the written method.

If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method.

Consider where exchanged digits are placed and make sure this is consistent.

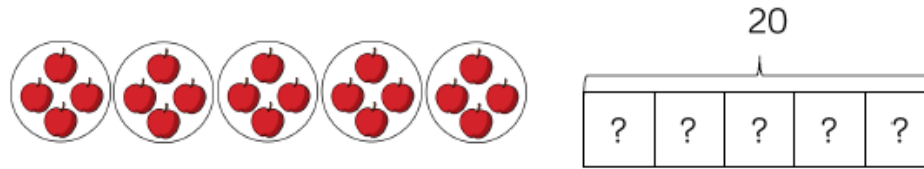


Priory
Primary School

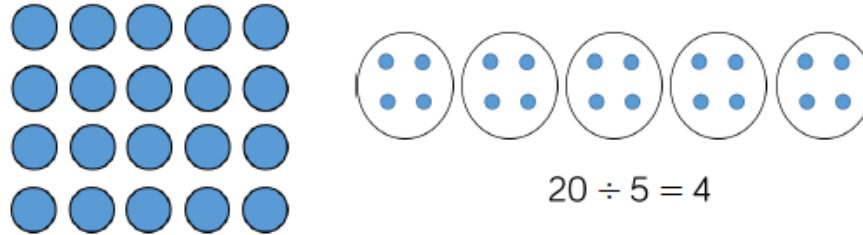
Division

Skill: Solve 1-step problems using multiplication (sharing)

Year: 1/2



There are 20 apples altogether.
They are shared equally between 5 bags.
How many apples are in each bag?



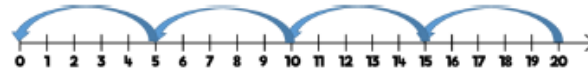
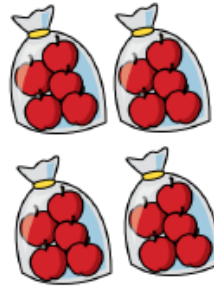
Children solve problems by sharing amounts into equal groups.

In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally.

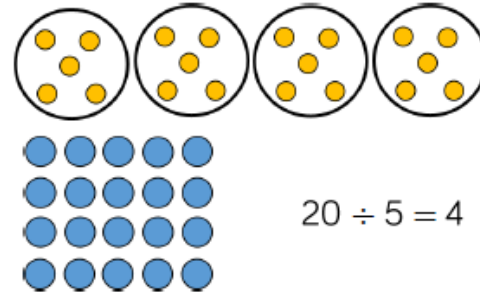
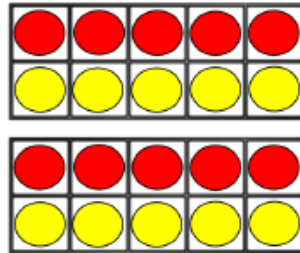
In Year 2, children are introduced to the division symbol.

Skill: Solve 1-step problems using division (grouping)

Year: 1/2



There are 20 apples altogether.
They are put in bags of 5.
How many bags are there?



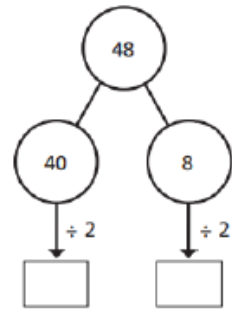
$$20 \div 5 = 4$$

Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division.

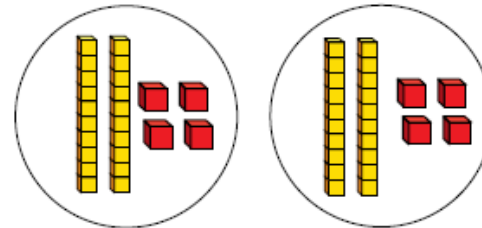
Skill: Divide 2-digits by 1-digit (sharing with no exchange)

Year: 1/2

Tens	Ones
10 10	1 1 1 1
10 10	1 1 1 1



$$48 \div 2 = 24$$



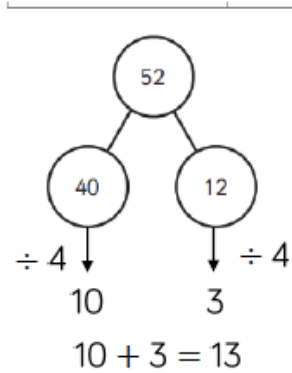
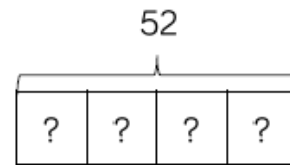
When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones.

Straws, Base 10 and place value counters can all be used to share numbers into equal groups.

Part-whole models can provide children with a clear written method that matches the concrete representation.

Skill: Divide 2-digits by 1-digit (sharing with exchange)

Year: 3/4



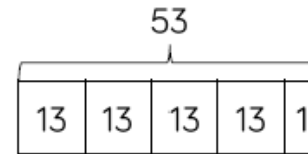
$$52 \div 4 = 13$$

When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones. Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

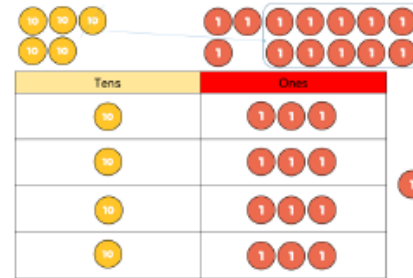
Flexible partitioning in a part-whole model supports this method.

Skill: Divide 2-digits by 1-digit (sharing with remainders)

Year: 3/4



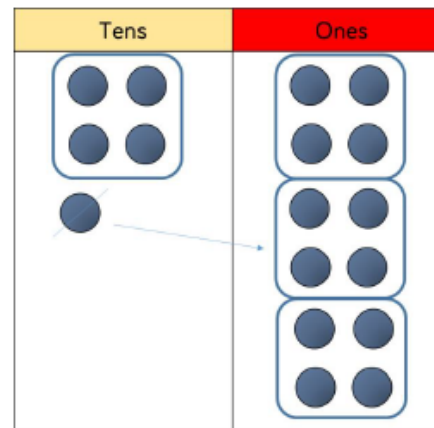
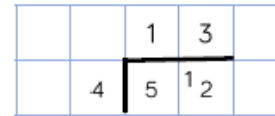
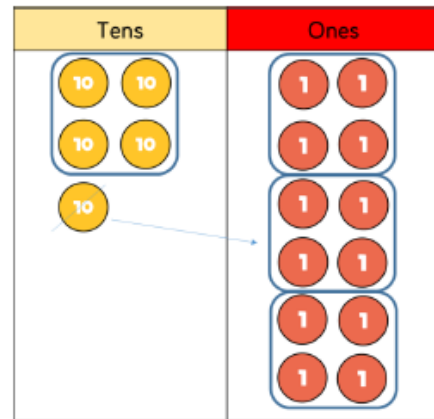
$$53 \div 4 = 13 \text{ r}1$$



When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones. Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made. Flexible partitioning in a part-whole model supports this method.

Skill: Divide 2-digits by 1-digit (grouping)

Year: 4/5



$$52 \div 4 = 13$$

When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor.

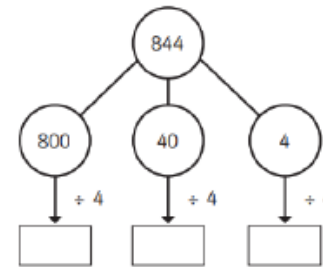
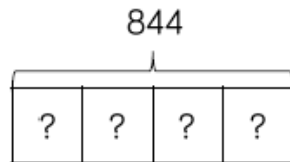
Language is important here. Children should consider 'How many groups of 4 tens can we make?' and 'How many groups of 4 ones can we make?'

Remainders can also be seen as they are left ungrouped.

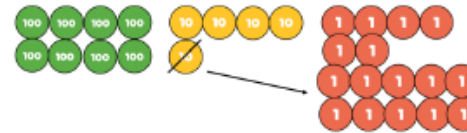
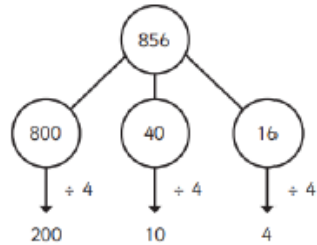
Skill: Divide 3-digits by 1-digit (sharing)

Year: 4

$$844 \div 4 = 211$$



$$844 \div 4 = 211$$



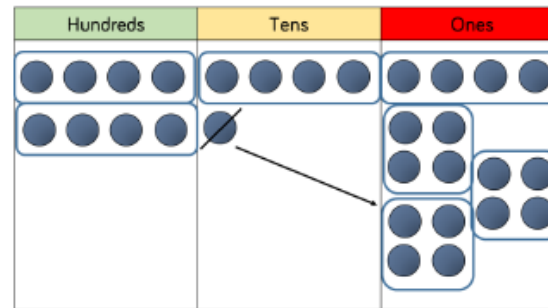
Children can continue to use place value counters to share 3-digit numbers into equal groups. Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows. This method can also help to highlight remainders. Flexible partitioning in a part-whole model supports this method.

Skill: Divide 3-digits by 1-digit (grouping)

Year: 5



		2	1	4
4	8	5	16	



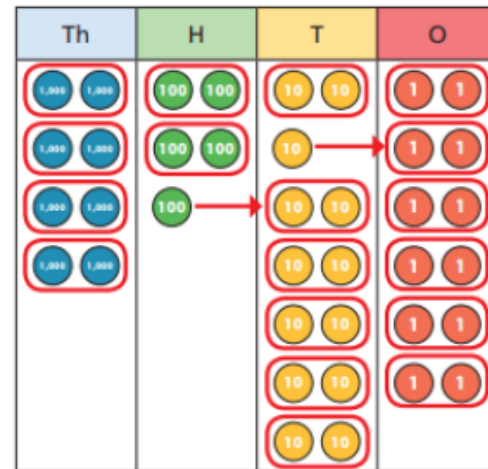
$$856 \div 4 = 214$$

Children can continue to use grouping to support their understanding of short division when dividing a 3-digit number by a 1-digit number.

Place value counters or plain counters can be used on a place value grid to support this understanding. Children can also draw their own counters and group them through a more pictorial method.

Skill: Divide 4-digits by 1-digit (grouping)

Year: 5



	4	2	6	6
2	8	5	13	12

$$8,532 \div 2 = 4,266$$

Place value counters or plain counters can be used on a place value grid to support children to divide 4-digits by 1-digit. Children can also draw their own counters and group them through a more pictorial method.

Children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges.

Skill: Divide multi digits by 2-digits (long division)

Year: 6

$$372 \div 15 = 24 \text{ r}12$$

			2	4	r	1	2
1	5	3	7	2			
	-	3	0	0			
			7	2			
	-		6	0			
			1	2			

- 1 × 15 = 15
- 2 × 15 = 30
- 3 × 15 = 45
- 4 × 15 = 60
- 5 × 15 = 75
- 10 × 15 = 150

When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction. This will depend on the context of the question.

Children can also answer questions where the quotient needs to be rounded according to the context.

			2	4	$\frac{4}{5}$
1	5	3	7	2	
	-	3	0	0	
			7	2	
	-		6	0	
			1	2	

$$372 \div 15 = 24 \frac{4}{5}$$

Glossary

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Numbers can be multiplied in any order.

Dividend – In division, the number that is divided.

Divisor – In division, the number by which another is divided.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make a product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Quotient – The result of a division

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor

Glossary

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative - numbers can be added in any order.

Complement - in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference - the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange - Change a number or expression for another of an equal value.

Minuend - A quantity or number from which another is subtracted.

Partitioning - Splitting a number into its component parts.

Reduction - Subtraction as take away.

Subitise - Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total - The aggregate or the sum found by addition.